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THE DYNAMICS OF A BRAKE SHOE AND "IMPACT GENERATED BY FRICTION"†

V. A. SAMSONOV
Moscow
email: samson@imec.msu.ru

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The interaction between a wheel and a brake shoe is discussed. It is noted that there is a critical set of parameters for which the friction force can take an arbitrary value. The phenomenon of "impact generated by friction" occurs in the system in the subcritical domain of the parameters. Versions of the post-impact behaviour of the bodies are described; one of them, the stopping of the wheel and the recoil of the brake shoe, corresponds to the process scenario described by Bolotov. It is also shown that it is possible for the wheel to rotate in the opposite direction.

It is commonly assumed that the initial conditions for the motion (at the instant of time t_0) of a system of connected bodies do not arise "on their own" but are created by external circumstances, which cease to act at the instant of time $t_0 - 0$ (immediately preceding the beginning of the motion). These circumstances not only specify the initial arrangement and velocity distribution of the parts of the system but also certain values R_- of the reactions of the constraints which define the system when $t \geq t_0$. In the functioning of many technological units, it is found that the quantities R_- are not identical to the values of the reactions which are necessary to maintain basic constraints. In classical problems (particularly, in the case of ideal bilateral constraints) this difference has usually been neglected while tacitly assuming that the reactions change at the instant of time t_0 in a jump-wise manner and at once take the necessary values.

However, in 1895, Painlevé [1] discovered that, in the case of systems with dry friction, the problem of determining the reactions may either have no solution or have several solutions. In both cases, the question of the nature of the motion of the system requires additional study. During the first two decades of the twentieth century these questions were discussed by mathematicians in the French and German scientific press. The work of the Russian mathematician, Moscow University Professor Ye. A. Bolotov [2], in which it was demonstrated that a system experiences "impact generated by friction"‡ in the first of the above-mentioned cases, apparently remained unknown to the participants in these discussions.

1. CONTACT WITHOUT EXTERNAL FORCES

Suppose a wheel of radius R (body A in Fig. 1) rotates around a fixed axis O_1 at a certain angular velocity ω_0. A rectangular brake pad B, which can also rotate about a fixed axis O_2 (with a unilateral constraint), leans against the wheel. If the brake shoe is not in contact with the wheel, it can rotate in any direction but, when it makes contact with the wheel, it can only rotate away from the wheel.

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‡ Professor Bolotov's work was highly appreciated by N. Ye. Zhukovskii, who not only gave it an honorable mention but also draw attention to it in his famous jubilee speech "Mechanics at Moscow University over the Last Half Century". The publishers of the Russian translation of Painlevé's book added material concerning discussions which had taken place abroad on the subject but, unfortunately, did not refer to the more profound results (in Zhukovskii's opinion) obtained by Bolotov. As a result Bolotov's achievements have remained unrecognized not only by the broader scientific community but some present-day researchers.

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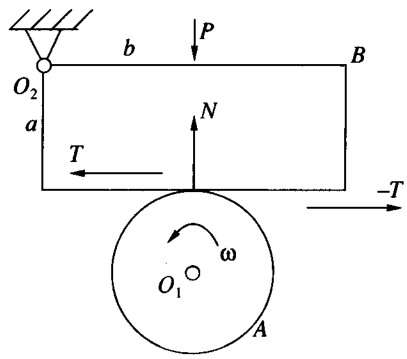

Fig. 1

Suppose that, initially, there are no external forces of any kind. Experience suggests that the interaction between the brake shoe and the wheel must lead to retardation of the wheel, that is, to a decrease in the angular velocity ω of its rotation. We will not consider what the mechanical-mathematical model gives if it is assumed that the bodies A and B are absolutely rigid.

When contact is maintained with the wheel, body B is in equilibrium under the action of two forces: the force $\mathbf{N}$ of the normal pressure from the side of the wheel and a dry sliding friction force $\mathbf{T}$. Hence,

$$aT - bN = 0 \tag{1.1}$$

where a and b are the arms of the lines of action of the forces with respect to the O_2 axis.

For the rotation of body A, we have

$$I_w\dot{\omega} = -RT \tag{1.2}$$

Here, I_w is the moment of inertia of the wheel about the O_1 axis.

In order to close the system of equations (1.1) and (1.2), it is necessary to invoke additional information concerning the relations between N and T. Since the wheel slides over the brake shoe, we use the empirical law for dry sliding friction (Coulomb's law)

$$T = fN \tag{1.3}$$

where f is the constant coefficient of friction. Moreover, the condition for the unilateral constraint to be maintained has the form

$$N \geq 0 \tag{1.4}$$

Substituting expression (1.3) into equality (1.1), we obtain the equation

$$(fa - b)N = 0 \tag{1.5}$$

From this, we derive that, in the case of the special (critical) set when

$$fa = b$$

an arbitrary value of N serves as a solution of Eq. (1.4). Consequently, the force T is also arbitrary, that is, it is impossible to reveal the law for the braking of the wheel using Eq. (1.2)!

Thus, a *first difficulty*, to which sufficient attention has not been paid, manifests itself.

When $fa \neq b$, we have $N = 0$ and $T = 0$. The wheel rotates at a constant velocity ω_0 and the brake show is located close to it with an infinitesimal gap. This state of motion, apparently, does not contradict common sense.

Moreover, the thesis "If two bodies under given conditions do not exert a pressure on one another, being absolutely smooth, then they also do not exert pressures when their surfaces are rough" was used by Painlevé and others.

The dependence of N on the parameter fa/b, according to Eq. (1.5) is shown in Fig. 2(a). The "intersection of two lines" which is shown here does not occur in a number of so-called "rough" geometrical forms and inevitably breaks down when any "Perturbation" is introduced into the system of equations (1.1)–(1.3).

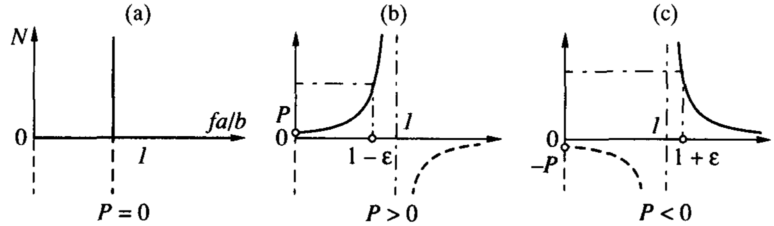

Fig. 2

2. A SMALL EXTERNAL FORCE

We will no "perturb" the system with an additional external force **P** (Fig. 1) which, for simplicity, acts along the same line as **N**. The left-hand side of the equilibrium equation of body B (1.1) is then supplemented with a term bP which, when account is taken of the friction law (1.3), gives

$$N = \frac{b}{b - fa}P \tag{2.1}$$

First, since the reference solution $N = 0$ held almost everywhere when $P = 0$, it seems natural to expect that we shall have a small N in the case of a sufficiently small value of P. We will now verify this assumption using formula (2.1). First, when $P > 0$ and the additional force clamps the brake show onto the wheel, the solution of Eq. (2.1) is found to be compatible with condition (1.4) only when $fa < b$ (Fig. 2b) and, when $fa > b$, the equilibrium equation for the brake show does not have a solution which is compatible with (1.4). Thus, a *second difficulty* manifests itself.

Second, for $P < 0$, when the additional force detaches the brake shoe from the wheel, Eq. (2.1) has a solution which is compatible with Eq. (1.4) only in the subcritical domain of the parameters, that is, when $fa > b$ (Fig. 2c). Thus, a *third difficulty* is revealed, which is often called a "paradox": friction does not contribute to the detachment of the brake shoe from the wheel, although the possibility of such a detachment is "obvious".

Third, in the case of the critical set of parameters, when $fa = b$, the equilibrium equation does not have a solution in general! (This is the *fourth difficulty*!)

Against the background of the above difficulties, we also point out that the assumption which has been made concerning the smallness of N is obviously not satisfied. For any (small) P, an ε-neighbourhood of the critical values of the parameters exists such that $N \gg P$. In this sense, it can be assumed that the fourth difficulty is resolved in the following manner: of the undermined set of N values when $P = 0$, a mechanical system selects just one: $N = +\infty$.

Since the time of Painlevé, it has been customary to consider the above-mentioned difficulties as being "mathematical", in the sense that mathematical model is insufficiently rigorous. However, comparison of (Figs 2a and b) suggests that the mechanical system changes qualitatively when the friction coefficient exceeds the critical value.

3. THE DISCONTINUITY OF THE REACTIONS

The discussion mentioned above transferred the problem into the realm of natural philosophy. In the case of the example being discussed, this problem can be formulated as follows. As long as there is a gap between the bodies, the wheel rotates at a constant velocity ω_0. We slowly bring the brake shoe up to the wheel by means of an external action and, at the instant of contact, we remove this external action (even though the force P remains). It is obvious that, at the instant of contact, the force N due to the pressure of the brake shoe on the wheel, is controllably small (in a thought experiment) even if it doesn't disappear completely. However, because of the properties of an absolutely rigid body, the force N is obliged to take a value immediately after the removal of the external action (instantaneously), which is ascribed to it by Eq. (2.1) (even in the case when $|fa - b| < b\varepsilon$ and the value of N is extremely large).

Of course, any difference between a real body and an absolutely rigid body transforms the change in the force exerted by the pressure (and the friction force at the same time) into a certain process which is highly short-term, but which sometimes cannot be completely neglected. Furthermore, the need to take account of such errors as the small, but nevertheless non-zero, rate of approach of the bodies is obvious in a real experiment.

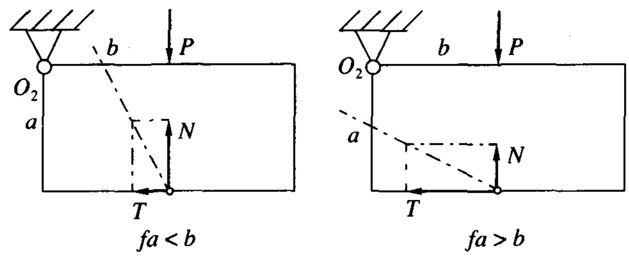

Fig. 3

The solution of the problem on a natural philosophical level was proposed almost 100 years ago by Bolotov [2] but has been undeservedly buried in oblivion.

We shall attempt to interpret Bolotov's reasoning as applied to the behaviour of a brake shoe. In order to do this, we set up a geometrical figure of the equilibrium of a brake shoe (Fig. 3).

In the case when $fa < b$, the overall force due to the action of the wheel on the brake shoe can balance the external force **P**. In the case when $fa > b$, the moment of the total force due to the action of the wheel on the brake shoe when $N > 0$ tends to turn it in the same direction as the moment of the force **P**. Equilibrium of the brake shoe is obviously impossible but its swivelling is hindered the wheel.

We now turn to the case when $fa < b$. Suppose the friction coefficient increases from experiment to experiment. While $fa < b$, an increase in the friction coefficient leads to a shortening of the braking time of the wheel in accordance with equality (1.2). Moreover, when $fa \to b - 0$, the braking time tends to zero.

A further increase in the friction coefficient (the case when $fa > b$) should shorten the braking time even further, but this is now impossible because any attempt to clamp the brake shoe to the wheel when $fa > b$ must lead to the instantaneous cessation of slippage. So, the wheel and the brake shoe experience "impact generated by friction"!

However, the impact of the bodies is far from always completed with the cessation of the relative motion of the colliding bodies, and post-impact motion of the bodies is quite possible. Within the framework of the ideas at this time regarding the two phases in the process of the collision of elastic bodies, Bolotov came to the conclusion which, in the case of the problem being considered, means that, after the wheel has stopped, the brake shoe "rebounds" from it and, here, the rate of rebound must be determined by the so-called coefficient of restitution.

4. ALLOWANCE FOR THE "NORMAL" PLIABILITY OF BODIES

It is pointless to discussion question of the collision of bodies without taking account of their deformability since the normal and shear forces (stresses) accompanying collisions, reach value such that any (real) bodies experience transient changes in shape. It is well known that, in the zone where there is intense sliding friction, all materials are heated and, for many of them, microdestruction of the surface occurs, leading to a change in the shape of the rubbing objects. In order to take these facts into accounts, it is necessary to introduce changes into Coulomb's law (1.3) itself. Of course, it is possible in principle here to formulate the corresponding contact problems in the mechanics of a deformable body but, for estimates, simplified models for taking account of the viscoelastic properties of the colliding bodies may also be suitable.

Within the framework of the problem being discussed, one such model was already being used at the beginning of the twentieth century, but it was applied to the problem of a brake shoe for the first time by Neimark and Fufayev in [4]. The idea consists of the inclusion of a certain viscoelastic element between the wheel and brake show which simulates some properties or other of the pliability and deformability of the bodies in the neighbourhood of the point of contact.

We shall pursue this idea while taking account of the fact that the final forces can be neglected in the study of impact interactions, that is, we put $P = 0$.

We will now simulate the so-called normal pliability of a brake shoe (Fig. 4). For this purpose, we place a viscoelastic spring with stiffness k and viscosity h, which are sufficiently large in magnitude, between the brake shoe and the wheel. We attach one end of the spring to the brake shoe and, to the other end, we attach a small body – the plate C of mass m, which, relative to the brake shoe, moves along the normal to the wheel and simulates the inertial properties of the deformed part of the brake

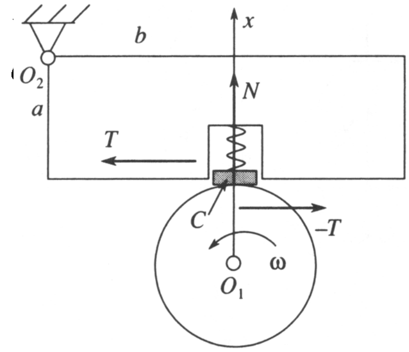

Fig. 4

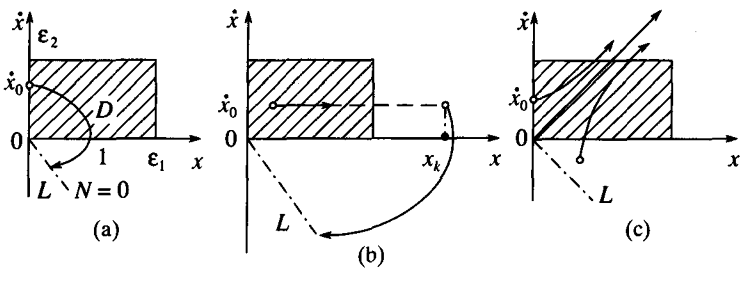

Fig. 5

shoe. Of course, the brake shoe B is turned to some extent during the interaction but, for simplicity, we shall assume that the orientation of the plate C is maintained.

The following forces act on the body C which has stopped after coming into contact with the wheel: a pressure **N** and friction **T** from the direction of the wheel (since $\omega \neq 0$, condition (1.3) is satisfied) and a force $F = -kx - h\dot{x}$ from the direction of the spring, where x is the magnitude of the contraction of the spring which is proportional to the angle of rotation of the brake shoe. From this, we have

$$N = kx + h\dot{x}, \quad T = f(kx + h\dot{x}) \tag{4.1}$$

and, for the rotation of the brake shoe, we obtain

$$I\frac{\ddot{x}}{b} = -Nb + Ta = (fa - b)(kx + h\dot{x})) \tag{4.2}$$

where I is the moment of inertia of the brake shoe about the O_2 axis.

We will consider versions of the behaviour of the solutions of Eq. (4.2).

Case A. When $fa < b$ (low friction), this equations corresponds to a traditional object in the oscillation theory, that is, to an oscillator, the motion of which must decay exponentially. The phase trajectory in the x, $\dot{x}$ plane is shown in Fig. 5. The above-mentioned external circumstances ensure the following initial conditions: $0 < x_0 < \varepsilon_1$, $0 < \dot{x}_0 < \varepsilon_2$, where ε_1 and ε_2 are relatively small. The domain D of possible initial condition is indicated by a dashed line in Fig. 5. A representative point, by moving along the phase trajectory 1 will reach the line L on which the condition $kx + h\dot{x} = 0$ is satisfied or $N = 0$. At this moment, the brake shoe acquires a small angular velocity $\dot{x}/b$ of rotation from the wheel and the contact of the body C with the wheel is destroyed.

This result corresponds completely to the left-hand side of Fig. 3 when $P = 0$. It is seen that, if the forces **N** and **T** are non-zero, they repel the brake shoe from the wheel.

The angular velocity of the wheel changes only slightly after this short contact time.

Case B. In the case of the critical set of parameters ($fa = b$) Eq. (5.2) degenerates into

$$I\ddot{x} = 0 \tag{4.3}$$

In this case, each point of the x axis in the $x, \dot{x}$ phase plane (Fig. 5b) corresponds to a state of rest, that is, the magnitude of N can actually take an arbitrary constant value depending on the initial deformation of the spring. This correlates fully with the form of the graph in Fig. 2(a).

However, if $\dot{x}_0 > 0$ (even if small), then $x = \dot{x}_0 t$, that is, the magnitude of the deformation of the spring increases with time. Together with this, the magnitude of the friction force also increases (see (4.1)) which leads to a slowing down of the rotation of the wheel and to it stopping. Suppose that, at the instant it stops, the x coordinate has reached a value x_k (Fig. 5b). Since the wheel is no longer rotating, $T = 0$ and the motion of the brake shoe is described by the equation of an oscillator

$$I\frac{\ddot{x}}{b} = -b(kx + h\dot{x}) \tag{4.4}$$

The phase trajectory of the unloading of the oscillator (4.4) is shown in Fig. 5(b). It is clear that the velocity with which the brake shoe rebounds from the wheel will be significantly higher than in case A, since a significant part of the rotational energy of the wheel has been expended in deforming the spring.

Case C. Suppose $fa > b$. Equation (4.2) can now be called an "anti-oscillator" since the position part of the force becomes repulsive and the velocity part becomes anti-dissipative. The phase trajectories for this case are shown in Fig. 5(c). Both x and $\dot{x}$ increase exponentially along the branch of the separatrix emerging from the origin of coordinates. This leads to an even faster increase in the friction force T than in case B and to a faster stopping of the wheel. After the wheel has stopped, the motion of the brake shoe is described by Eq. (4.4), and the phase trajectory of this motion is similar to that shown in Fig. 5(b) with the sole difference that it starts not only at a relatively large value of x_k but, also, at a rather large value of $\dot{x}_k$. A significant increase in the velocity of the post-impact rebound of the brake shoe is therefore to be expected.

We will now compare the conclusions drawn from an analysis of cases B and C and the above mentioned (Section 1) seemingly natural thesis of Painlevé. It can be seen that, when $fa \geq b$, the fixed point $x = 0, \dot{x} = 0$ exists, that is, a motion is theoretically possible for which the bodies do not develop a pressure on one another, but it is unstable and "impact generated by friction" is most likely under these conditions.

Remarks. 1. Note that, in the case when $fa > b$, Eq. (1.5) has a *unique* solution, that is, "impact generated by friction" is possible and the equations of motion of the solid bodies then "do not promise any paradoxes". This should be borne in mind in the case of systems with bilateral constraints since each of the sides can be found to be prestressed and, hence, "impact generated by friction" can be produced not by the side of a bilateral constraint, which appears to be loaded by the external forces, but by the opposite side.

2. The course of the impact and its completion are similar to that described by Bolotov for a more complex problem.

5. ALLOWANCE FOR "TANGENTIAL" PLIABILITY

Of course, normal pliability, when there is a tangential load, cannot correspond to the mechanical properties of all materials from which brake shoes can be manufactured. It is most likely that this is one of the limiting properties of materials. Obviously, at the other limit, there is the property of tangential pliability, to simulate which we shall also borrow the scheme proposed earlier in [4], having slightly corrected it.

Suppose now (Fig. 6) that a body C of small mass m can only move along a tangent to the wheel. Between the brake shoe and the body C we place a viscoelastic spring with stiffness and viscosity k and h.

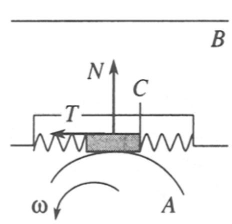

Fig. 6

The equation of motion of the body C, the position of which relative to the brake shoe we define by the coordinate x, has the form

$$m\ddot{x} = -kx - h\dot{x} + T \tag{5.1}$$

The equation for the equilibrium of the brake shoe is

$$a(kx + h\dot{x}) = bN \tag{5.2}$$

As long as $\dot{x} < \omega R$, we have $T = fN$ and Eq. (5.1) takes the form

$$m\ddot{x} = \left(f\frac{a}{b} - 1\right)(kx + h\dot{x}) \tag{5.3}$$

This equation is now analogous to Eq. (4.2) which has been discussed. Without repeating the details, we will concentrate our attention solely on the case when $fa > b$. Immediately after the brake shoe comes into contact with the wheel, initial conditions will inevitably be created for which the phase trajectory (Fig. 5c) of Eq. (5.3) is close to the branch of the separatrix emerging from the origin of coordinates. Since, $m \ll I/b^2$, the velocity of the body C after a short interval of time is equal to the angular velocity of the points of the wheel, that is, slipping ceases and

$$\dot{x} = \omega_* R \tag{5.4}$$

As a consequence of the small mass m of the body C, the magnitude of ω_* is only slightly different from ω_0 and, therefore, in the next phase of the motion, there is intense compression of the spring in accordance with the equation for the rotation of the wheel which, when there is no slipping, that is, when $\dot{x} = \omega R$, takes the form

$$(I + mR^2)\frac{\ddot{x}}{R} = -R(kx + h\dot{x}) \tag{5.5}$$

which is analogous to Eq. (4.4). The initial conditions of compression for the oscillator (5.3) correspond to practically the whole of the initial energy of rotation of the wheel (although the magnitude of x_k may be found to be rather small). Taking account of this, we conclude that the phase trajectory of Eq. (5.5) behaves as in Fig. 5(b), it can easily be verified that the condition $T < fN$, which guarantees that there is no slipping, is satisfied in this segment. It is natural that contact of the wheel with the brake shoe is broken on reaching the representative point of the line L.

We will now consider this question in greater detail. First, since $\dot{x} < 0$ at the instant of detachment, the wheel now rotates in the reverse direction! The magnitude of the velocity of this reverse rotation is comparable with ω_0. This result serves as a supplement to the above-mentioned conclusion of Bolotov. Second, the system of the brake shoe plus the body C acquires a certain (although small) angular momentum with respect to the O_2 axis. Hence, this system will start to rotate slowly about the O_2 axis in the direction of the wheel. This rotation is non-uniform while the body C performs oscillations with respect to the brake shoe.

6. INTERIM SUMMARY

It can be assumed that the account in Sections 4–6 enables one to explain Fig. 2.

Actually, the segment of the line $N = 0$ on which $fa < b$, as a whole inherits the same property which the system possessed in the case of a small value of f: after a short time of contact of the wheel, the brake shoe acquires a small angular velocity but the rotation of the wheel in the initial direction is maintained.

At the critical value of the friction coefficient $f = b/a$ (or for greater values) a jump-like, "catastrophic" change in the above-mentioned property occurs. Contact of the brake shoe with the wheel leads in the first phase of the contact (pointed out by Bolotov) to an intense reduction in the rate of slippage. In the second phase, there is a matching increase in the deformation of the contacting bodies both along the normals to their surfaces as well as in a tangential direction. Finally, in the third phase of the unloading of the deformed bodies, the latter, depending on the properties of the contacting bodies, acquire post-impact velocities which break the contact between the brake shoe and the wheel.

Two limiting cases of the pliability of bodies have been considered above. Jointly taking account of the normal and tangential pliabilities naturally leads to a larger variety of versions of the post-impact behaviour of bodies.

7. DETACHMENT OF THE BRAKE SHOE

It can be shown that the explanation of the process of impact generated by friction is of a comprehensive character, especially as an attempt to clamp the brake shoe to the wheel by a force P leads to the appearance on the right-hand sides of Eqs (4.2), (4.4), (5.2) and (5.3) of positive terms, which are equivalent to a displacement of the initial and final points along the phase trajectory shown in Fig. 5(c).

However, there is still a third feature of formula (2.1) and Fig. 2(c), which is associated with a negative value of the force P or, more accurately, with the change in the direction of the small external force applied to the brake shoe: in this case formula (2.1) gives a certain finite value for N in the domain $fa > b$. Consequently, the force T also takes a finite value and retardation of the wheel occurs with a bounded (and, when $P_1 = -P > 0$, even a small) acceleration, almost as was the case in the domain $fa < b$. Again, we encounter a "catastrophic" change in a certain property. When $P < 0$ and $fa < b$, it follows from formula (2.2) that $N < 0$, that is, equilibrium of the brake shoe is impossible and the external force detaches it from the wheel!

We will now only consider the case of tangential pliability. Since the equilibrium equation of the brake shoe takes the form

$$a(kx + h\dot{x}) = b(N + P_1) \tag{7.1}$$

Eq. (5.2) for the motion of the body C also changes

$$m\ddot{x} = \left(\frac{fa}{b} - 1\right)(kx + h\dot{x}) - fP_1 \tag{7.2}$$

In the phase plane $x, \dot{x}$ (Fig. 7), we now separate out the domain of motions which is of interest

$$x > 0, \quad N > 0 \Rightarrow kx + h\dot{x} > (b/a)P_1$$

It is clear that the origin of coordinates (the point $x = 0, \dot{x} = 0$) does not fall within this domain. In fact, if the spring is unstressed at the initial instant after contact, then the force P_1 turns the brake shoe from the wheel without any resistance.

If the brake shoe is clamped in order to ensure contact such that the spring will start to deform, then it can fall into the domain $N > 0$, where motion of the representative point along one of the phase trajectories (Fig. 7) occurs. The point

$$x_* = \frac{fbP_1}{(fa - b)k}, \quad \dot{x} = 0 \tag{7.3}$$

which is a fixed point of Eq. (8.1) (a state of rest), is a saddle point, that is, the representative point asymptotically approaches this fixed point along the two ingoing branches of the separatrix (the direction of motion of the representative point is indicated by the arrows in Fig. 7). Retardation of the wheel (which is relatively slow compared with that described in the preceding sections) occurs in parallel with this process. For all trajectories located below the ignoring branches of the separatrix, the representative point falls on the line $N = 0$, after which the brake shoe separates from the wheel which, up to this instant, can have hardly lost all of its rotation velocity.

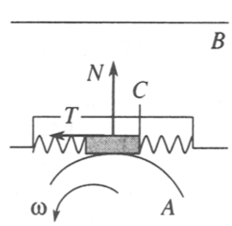

Fig. 7

Note that the motions in a small neighbourhood of the point $(x_*, 0)$ are accompanied by the small forces N and T. The introduction of some additional forces into the system can lead to a change in the character of the fixed point, such as converting it into a stable node, etc.

The possibility of motions in the neighbourhood of the branch of the separatrix entering from the fixed point is fundamentally new (compared with what has been described in Section 2). In the case of these motions, $\dot{x}$ and the difference $x - x_*$ increase exponentially. Of course, these properties are only preserved until $\dot{x} < \omega R$, that is, up to the instant when slipping ceases.

The scenario for the subsequent events only differs from that described in Section 6 in the fact that the process of the repeated occurrence of slippage, accompanying the reverse rotation of the wheel, begins somewhat earlier than the detachment of the brake shoe. This possibly leads to some reduction in the final velocity of the reverse rotation of the wheel compared with Section 6.

8. CONCLUSION

The above discussion enables us to sum up the so-called "resolution" of the paradoxed of dry friction as it applies to a specific mechanical system – a brake shoe plus a rotating wheel. The introduction of a viscoelastic element has enabled us to establish that, when $fa > b$, the phenomenon of "impact generated friction" occurs in the system, which requires independent study and simulation.

The intensity of the impact generated by friction (the difference in the velocities prior to impact) is far higher than the intensity of the collision of the bodies which is unavoidable when they are brought into contact. Only when the solution of the equations of the dynamics of absolutely rigid bodies is unique is there some likelihood of an impact-free breaking of the contact when there is a special arrangement of the initial contact loads.

The fundamental difference between impact generated by friction and the classical scheme for the collision of bodies lies in the existence of three (and not two) phases, namely, slippage, concerted deformation (loading of the constraints) and restoration (unloading).

However, we do not claim to have finally solved the difficulties revealed by Painlevé.

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